Equations for airborne gravity data processing

Parameters

- $a = 6378137$ m (Equatorial radius);
- $b = 6356752,3141$ m (Polar radius);
- $f = (a - b)/a = 0.00335281068$ (Earth flattening);
- $w = 7292115 \times 10^{-11}$ rad·sec^{-1} (Rotation rate);
- $\gamma_E = 9.7803267715$ m·sec^{-2} (Equatorial gravity);
- $\gamma_P = 9.8321863685$ m·sec^{-2} (Polar gravity);
- $e^2 = 0.00669438002290$ (Eccentricity squared).

Conversion and calibration factors

- $e_d = 1.11585 \times 10^5$;
- $n_d = 1.11369 \times 10^5$;
- 1 m·sec^{-2} = 10^5 Gal;
- $k_f = 0.9899$ (S-80 gravity meter scale factor);
- $k_b = 30$ mGal·mV^{-1} (S-80 gravity meter beam derivative factor at 1 Hz);
- $k_{xa} = 25.110$ mGal·mV^{-1} (S-80 gravity meter cross accelerometer factor);
- $k_{la} = 26.483$ mGal·mV^{-1} (S-80 gravity meter long accelerometer factor).

Fundamental gravity equation

$$g = k_f \cdot (ST + k_b \cdot \dot{B} + CC) \tag{1}$$

where

- g is the relative gravity measured by the gravity meter;
- ST is the spring tension value;
- $\dot{B}$ is the beam derivative;
- CC are the cross-coupling corrections.

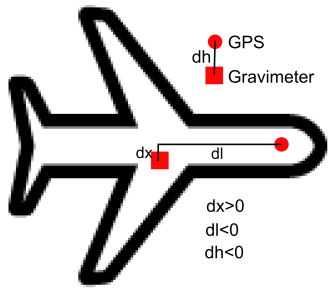

Figure 1: Leverarm correction.

Lever arm correction

$$\delta\lambda = \frac{dl \cdot \sin(\theta) + dx \cdot \cos(\theta)}{e_d \cdot \cos(\phi_0)} \quad (2)$$

$$\delta\phi = \frac{dl \cdot \cos(\theta) - dx \cdot \sin(\theta)}{n_d} \quad (3)$$

$$\delta H = dh + dl \cdot \sin(\alpha) + dx \cdot \sin(\beta) \quad (4)$$

$$\phi = \phi_0 + \delta\phi \quad (5)$$

$$\lambda = \lambda_0 + \delta\lambda \quad (6)$$

$$H = H_0 + \delta H \quad (7)$$

where

- dl, dx and dh are described in Fig.1;
- θ is the airplane heading;
- ϕ_0 is the GPS latitude;
- λ_0 is the GPS longitude;
- H_0 is the GPS elevation;
- α is the pitch angle;
- β is the roll angle.
- ϕ is the gravity meter latitude;
- λ is the gravity meter longitude;
- H is the gravity meter elevation.

Kinematic parameters

$$C_N = \frac{a}{\sqrt{1 - e^2 \cdot \sin^2(\phi)}} \tag{8}$$

is the radius of curvature for equatorial meridian and

$$C_M = \frac{a \cdot (1 - e^2)}{[1 - e^2 \cdot \sin^2(\phi)]^{3/2}} \tag{9}$$

is the radius of curvature for prime meridian. The East and West velocites are

$$V_E = (C_N + H) \cdot \cos(\phi) \cdot \dot{\lambda} \tag{10}$$

$$V_N = (C_M + H) \cdot \dot{\phi} \tag{11}$$

respectively, and the vertical velocity is

$$V_U = \dot{H} \tag{12}$$

The East and west accelerations are

$$A_E = \dot{V}_E + \left(\frac{V_E}{C_N + H} + 2w\cos(\phi) \right) \cdot (V_U - V_N \tan(\phi)) \tag{13}$$

$$A_N = \dot{V}_N + \left(\frac{V_E}{C_N + H} + 2w\cos(\phi) \right) \cdot \tan(\phi) \cdot V_E + \frac{V_N \cdot V_U}{C_M + H} \tag{14}$$

The unitary vectors along the flightline are given by

$$e_x = \sin(\theta) \tag{15}$$

$$e_y = \cos(\theta) \tag{16}$$

so that the cross and long accelerations are given by

$$x_{acc} = e_x \cdot A_N - e_y \cdot A_E \tag{17}$$

$$l_{acc} = e_x \cdot A_E + e_y \cdot A_N \tag{18}$$

Absolute gravity correction

$$\delta g_{abs} = g^0_{abs} - k_f \cdot ST_0 \tag{19}$$

where g^0_{abs} is the absolute gravity at the airport and ST_0 is the spring tension value of the gravity meter still reading at the airport.

Vertical acceleration correction

$$\delta g_{AU} = \ddot{H} = \dot{V}_U \tag{20}$$

is the vertical acceleration correction.

Eotvos correction

$$\delta g_{EOT} = \frac{V_N^2}{a} \cdot \left\{1 - \frac{H}{a} + f \cdot [2 - 3\sin^2(\phi)]\right\} + \frac{V_E^2}{a} \cdot \left[1 - \frac{H}{a} - f \cdot \sin^2(\phi)\right] + 2wV_E \cdot \cos(\phi) \quad (21)$$

Tilt correction

$$\delta g_{tilt} = \sqrt{a_x^2 + a_y^2 + g^2 - A_E^2 - A_N^2} - g \sim \frac{a_x^2 + a_y^2 - A_E^2 - A_N^2}{2g} \quad (22)$$

where a_x and a_y are the cross and long accelerometer components calibrated by using the scale factors k_{xa} and k_{la}. An alternative expression bias-free is given by

$$\delta g_{tilt} = (1 - \cos(phi_x) \cdot \cos(\phi_y)) \cdot g + \sin(\phi_x) \cdot a_x + \sin(\phi_y) \cdot \cos(\phi_x) \cdot a_y \quad (23)$$

where $\phi_x \simeq (a_x - x_{acc})/g$ and $\phi_y \simeq (a_y - l_{acc})/g$, respectively.

Latitude correction

$$\delta g_{lat} = -9.7803267715 \cdot \frac{1 + 0.00193185138639 \cdot \sin^2(\phi)}{\sqrt{1 - e^2 \cdot \sin^2(\phi)}} \quad (24)$$

Free-air correction

$$\delta g_{fa} = 0.3086 \cdot H \quad (25)$$

Gravity anomaly (free-air)

$$\Delta g_{fa} = g + \delta g_{abs} + \delta g_{AU} + \delta g_{EOT} + \delta g_{tilt} + \delta g_{lat} + \delta g_{fa} \quad (26)$$